C4 JAN 09

1) $y^2-3y=x^3+8$

$\frac{d}{dx}y^2-\frac{d}{dx}3y=\frac{d}{dx}x^3+\frac{d}{dx}8 \Rightarrow 2y\frac{dy}{dx}-3\frac{dy}{dx}=3x^2$

$\Rightarrow \frac{dy}{dx}=\frac{3x^2}{2y-3}$

$y=3 \Rightarrow 9-9=x^3+8 \Rightarrow x^3=-8 \Rightarrow x=-2$

gradient $=\frac{dy}{dx}=\frac{3(-2)^2}{2(3)-3}=\frac{12}{3}=4.$

2) $\int_0^2 \frac{3}{(1+4x)^{\frac{1}{2}}}dx$

$u=(1+4x)^{\frac{1}{2}}$ $\times 1.5$

$\frac{du}{dx}=\frac{4}{2}(1+4x)^{-\frac{1}{2}} \Rightarrow \times 1.5$

$=\int_0^2 3(1+4x)^{-\frac{1}{2}}$

$=\left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2=\left(\frac{3}{2}\times 9^{\frac{1}{2}}\right)-\left(\frac{3}{2}\times(1)^{\frac{1}{2}}\right)$

$=\underline{3}$

b) Vol $=\pi\int_0^2 y^2dx=\pi\int_0^2\frac{9}{1+4x}dx=\frac{9}{4}\pi\int_0^2\frac{4}{1+4x}dx$

$=\frac{9}{4}\pi\left[\ln(1+4x)\right]_0^2=\frac{9}{4}\pi(\ln 9-\cancel{\ln 1})$

$=\frac{9}{4}\pi\ln 3^2 = \frac{9}{2}(\pi\ln 3)$

3) $27x^2+32x+16=A(3x+2)(1-x)+B(1-x)+C(3x+2)^2$

$x=1 \Rightarrow 75=C(5)^2 \Rightarrow \underline{C=3}$

$x=-\frac{2}{3} \Rightarrow \frac{20}{3}=\frac{5}{3}B \Rightarrow \underline{B=4}$

$x=0 \Rightarrow 16=2A+B+4C \Rightarrow 2A=0 \Rightarrow A=0$

b) $f(x)=4(3x+2)^{-2}+3(1-x)^{-1}$

$f(x)=\cancel{4}\times \cancel{2^{-2}}(1+\frac{3}{2}x)^{-2}+3(1-x)^{-1}$

$f(x)=\left(1+(-2)\left(\frac{3}{2}x\right)+\frac{(-2)(-3)}{2}\left(\frac{3}{2}x\right)^2\right)+3\left(1+(-1)(-x)+\frac{(-1)(-2)}{2}(-x)^2\right)$

$$\left(1-3x+\frac{27}{4}x^2\right)+\left(3+3x+3x^2\right) = 4+\frac{39}{4}x^2$$

c) $f(0.2) = 4.341715976$

$f(0.2) \simeq 4.39 \Rightarrow \text{error} = 0.048284024$

$$\%\text{error} = \frac{0.048284024}{4.341715976} \times 100 = \underline{1.1\%}$$

4) l_1 and l_2 perp $\Rightarrow \begin{pmatrix}-2\\1\\-4\end{pmatrix}\cdot\begin{pmatrix}q\\2\\2\end{pmatrix} = 0 \Rightarrow -2q+2-8=0$

$2q=-6 \Rightarrow \underline{q=-3}$

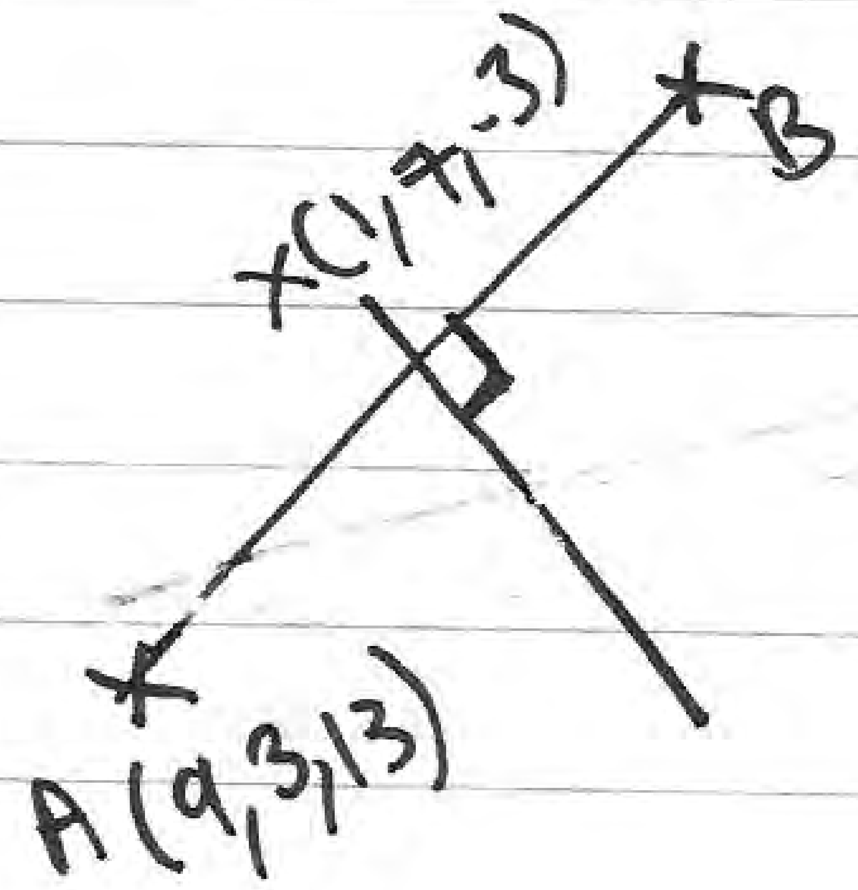
b) $\begin{pmatrix}11-2\lambda\\2+\lambda\\17-4\lambda\end{pmatrix} = \begin{pmatrix}-5-3\mu\\11+2\mu\\p+2\mu\end{pmatrix}$ $\quad 2\lambda-3\mu=16$, $\lambda-2\mu=9 \Rightarrow 2\lambda-3\mu=16$, $\underline{2\lambda-4\mu=18}$ $-$

$\underline{\mu=-2}$ $\quad \underline{\lambda=5}$

$17-4(5) = p+2(-2) \Rightarrow p = 17-20+4 \Rightarrow \underline{p=1}$

c) $\lambda=5 \Rightarrow$ Intersect at $\begin{pmatrix}11-10\\2+5\\17-20\end{pmatrix} = \begin{pmatrix}1\\7\\-3\end{pmatrix}$ $X(1,7,-3)$

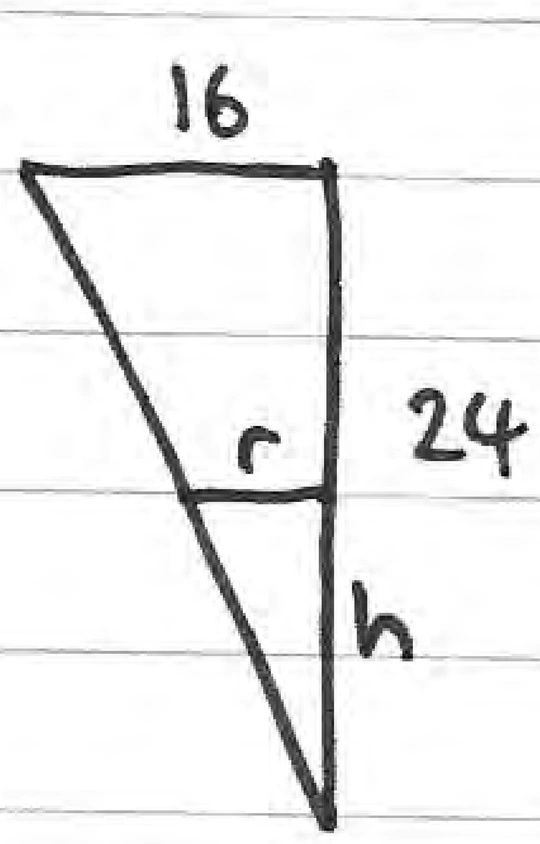
d)



$b = a + 2\overrightarrow{AX}$ $\quad \overrightarrow{AX} = x-a = \begin{pmatrix}1\\7\\-3\end{pmatrix}-\begin{pmatrix}9\\3\\13\end{pmatrix}$

$b = \begin{pmatrix}9\\3\\13\end{pmatrix}+2\begin{pmatrix}-8\\4\\-16\end{pmatrix} = \begin{pmatrix}-7\\11\\-19\end{pmatrix}$

5)



$\Rightarrow \frac{r}{h} = \frac{16}{24} \Rightarrow r = \frac{2}{3}h$

$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi\left(\frac{2}{3}h\right)^2 h = \underline{\frac{4}{27}\pi h^3}$

b) $\frac{dV}{dt} = 8$ $\quad \frac{dV}{dh} = \frac{4}{9}\pi h^2$ $\quad \frac{dh}{dt} = \frac{dV}{dt}\times\frac{dh}{dV}$

$$\frac{dh}{dt} = 8\left(\frac{1}{\frac{4}{9}\pi h^2}\right) = \frac{72}{4\pi h^2} = \frac{18}{\pi h^2} \qquad h=12 \Rightarrow \frac{dh}{dt} = 0.0398 \text{ cm/s}$$

$$= \frac{18}{144\pi}$$

6) $$\int \tan^2 x \, dx = \int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1-\cos^2 x}{\cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} dx = \int \sec^2 x - 1 \, dx$$

$$= \tan x - x + c$$

b) $$\int \frac{1}{x^3} \ln x \, dx \qquad u = \ln x \qquad v = -\frac{1}{2}x^{-2}$$

$$\frac{du}{dx} = \frac{1}{x} \qquad \frac{dv}{dx} = x^{-3}$$

$$= -\frac{\ln x}{2x^2} + \int \frac{1}{2x^2} \times \frac{1}{x} dx = -\frac{\ln x}{2x^2} + \frac{1}{2}\int \frac{1}{x^3} dx$$

$$= \frac{-\ln x}{2x^2} - \frac{1}{4}x^{-2} + c = -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + c$$

c) $$u = 1 + e^x \Rightarrow e^x = u - 1 \qquad e^{3x} = (e^x)^3 = (u-1)^3$$

$$\frac{du}{dx} = e^x \Rightarrow dx = \frac{du}{e^x} \Rightarrow dx = \frac{du}{u-1}$$

$$\Rightarrow \int \frac{e^{3x}}{1+e^x} dx = \int \frac{(u-1)^{\cancel{3}\,2}}{u} \frac{du}{\cancel{(u-1)}} = \int \frac{(u-1)^2}{u} du$$

$$\Rightarrow \int \frac{u^2 - 2u + 1}{u} du = \int u - 2 + \frac{1}{u} \, du$$

$$= \frac{1}{2}u^2 - 2u + \ln u + c$$

$$= \frac{1}{2}(1+e^x)^2 - 2(1+e^x) + \ln(1+e^x) + c$$

$$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$$

$$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) - \frac{3}{2} + c$$

$$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k$$

#

8) $x = t^3 - 8t$ $\quad y = t^2$ $\quad t=-1$ $\quad x = 7$ $\quad y = 1$ $\quad A(7,1)$

b) $\frac{dx}{dt} = 3t^2 - 8$ $\quad \frac{dy}{dt} = 2t$ $\quad \frac{dy}{dx} = \frac{2t}{3t^2-8}$ $\quad t=-1$ $\quad \frac{dy}{dx} = \frac{-2}{-5}$

$m_t = \frac{2}{5}$ $\quad y - 1 = \frac{2}{5}(x-7) \Rightarrow 5y - 5 = 2x - 14 \Rightarrow \underline{2x - 5y - 9 =}$

c) $\frac{dy}{dx} = \frac{2t}{3t^2-8} = \frac{2}{5} \Rightarrow 10t = 6t^2 - 16 \Rightarrow 3t^2 - 5t - 8 = 0$

$(3t-8)(t+1) = 0$

$t = \frac{8}{3}$

$x = \left(\frac{8}{3}\right)^3 - 8\left(\frac{8}{3}\right) = \frac{-64}{27}$ $\quad y = t^2 = \left(\frac{8}{3}\right)^2 = \frac{64}{9}$ $\quad B\left(\frac{-64}{27}, \frac{64}{9}\right)$

8) $x = t^3 - 8t$ $\quad y = t^2$ $\quad t = -1$ $\quad x = -1 + 8 = 7$ $\quad y = (-1)^2 = 1$ $\quad A(7,1)$

b) $\frac{dx}{dt} = 3t^2 - 8$ $\quad \frac{dy}{dt} = 2t$ $\quad \frac{dy}{dx} = \frac{2t}{3t^2-8}$ $\quad t=-1$ $\quad \frac{dy}{dx} = \frac{-2}{-5}$

$\Rightarrow m_t = \frac{2}{5}$ $\quad y - 1 = \frac{2}{5}(x-7) \Rightarrow 5y - 5 = 2x - 14$

$\Rightarrow \underline{2x - 5y - 9 = 0}$

c) $2(t^3 - 8t) - 5(t^2) - 9 = 0$

$\Rightarrow 2t^3 - 16t - 5t^2 - 9 = 0 \Rightarrow 2t^3 - 5t^2 - 16t - 9 = 0$

$(t+1)(2t^2 + At - 9)$ $\quad 2t^2 + At^2 \equiv -5t^2 \Rightarrow 2 + A = -5$

$\underline{A = -7}$

$(t+1)(2t^2 - 7t - 9)$

$(t+1)(2t-9)(t+1)$ $\quad t = -1,\ t = \frac{9}{2}$

$x = \left(\frac{9}{2}\right)^3 - 8\left(\frac{9}{2}\right) = \frac{441}{8}$ $\quad y = \left(\frac{9}{2}\right)^2 = \frac{81}{4}$ $\quad B\left(\frac{441}{8}, \frac{81}{4}\right)$